$\sin(x+y) = \sin x \sin y + \cos x \cos y$

$$f(x) = \frac{x - \sin(2x)}{\sin x}$$

a) define $f(0)$ so f is continuous.

$$\Rightarrow \lim_{x\to 0} \frac{x-\sin(2x)}{\sin x} = \lim_{x\to 0}\left(\frac{x}{\sin x} - \frac{\sin(2x)}{\sin x}\right)$$

$$= 1 - \lim_{x\to 0} \frac{\sin(2x)}{\sin(x)}$$

$$= 1 - \lim_{x\to 0} 2\frac{\sin(2x)}{2x} \cdot \frac{x}{\sin(x)}$$

$$= 1 - 2\cdot 1 \cdot 1 = -1$$

Rule

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$f(x)$ is continuous at $x=a$ if

* $f(x)$ is defined at $x=a$
* $\lim_{x\to a} f(x)$ exists
* $\lim_{x\to a} f(x) = f(a)$

$$f(x) = \frac{x - \sin(2x)}{\sin x}$$

b) with $f(0) = -1$, is $f(x)$ differentiable?

$$f'(a) = \lim_{x \to a} \underbrace{\frac{f(x) - f(a)}{x - a}}_{\text{slope of secant line}}$$

$$f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} \text{ if this exists}$$

$$= \lim_{x \to 0} \frac{\frac{x - \sin(2x)}{\sin x} - (-1)}{x} = \lim_{x \to 0} \frac{x - \sin(2x) + \sin x}{x \sin x}$$

intermediate step

$$\frac{\frac{x - \sin(2x)}{\sin x}}{x} = \frac{x - \sin(2x)}{x \sin x}$$

recall L'Hospital's rule:

if $\lim_{x \to a} \frac{f(x)}{g(x)}$ is an indeterminate form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$

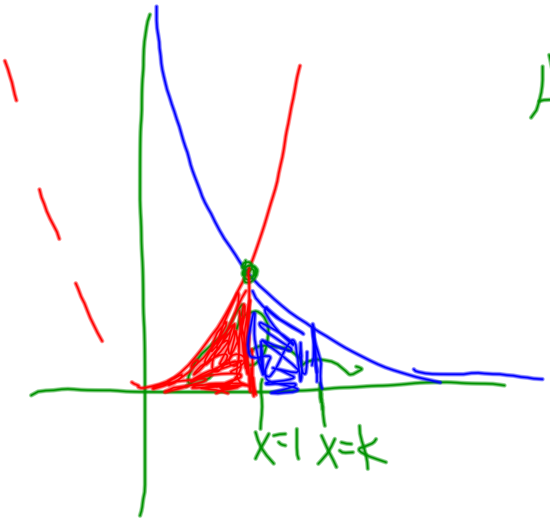
then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

$$\lim_{x \to 0} \frac{x - \sin(2x) + \sin(x)}{x \sin(x)} \overset{L'H}{=} \lim_{x \to 0} \frac{1 - 2\cos(2x) + \cos(x)}{\sin x + x \cos x} = \frac{0}{0}$$

$$\overset{L'H}{=} \lim_{x \to 0} \frac{+4\sin(2x) - \sin(x)}{\cos x + (\cos x - x \sin x)} = \frac{0}{2}$$

$$= 0$$

Under both $y=3x^2$, $y=\frac{3}{x}$, left of $x=k>1$



$$\text{Area} = \int_0^1 3x^2\,dx + \int_1^k \frac{3}{x}\,dx$$

$$= x^3\Big|_0^1 + 3\ln x\Big|_1^k$$

$$= (1-0) + 3(\ln k - \ln 1)$$

$$= 1 + 3\ln k$$

$$[\text{OR } 1+\ln k^3]$$

b) $7 = 1 + 3\ln k$

$2 = \ln k$

$e^2 = k$

c) $A = 1 + 3\ln k$ $\quad \frac{dA}{dt} = 5$; when $k=15$ what is $\frac{dk}{dt}$?

$$\frac{dA}{dt} = \frac{3}{k}\frac{dk}{dt}$$

$$5 = \frac{3}{15}\frac{dk}{dt} \Rightarrow \frac{dk}{dt} = 25$$

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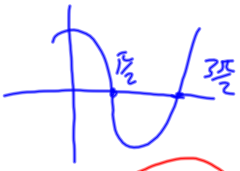
$f(x) = \cos^2 x + 2\cos x \qquad [0, ?]$

a) f(x)=0
b) minimum
c) concave up

(a)



$\frac{\pi}{2} \quad \frac{3\pi}{2}$

$\cos^2 x + 2\cos x = 0$

$(\cos x)(\cos x + 2) = 0$

$\cos x = 0$

$\cos x + 2$ impossible

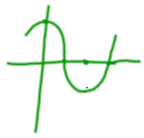
b) find minimum

$f'(x) = 2\cos(x)(-\sin(x)) - 2\sin(x)$

$-\sin(x)(2\cos(x) + 2) = 0$

$-\sin(x) = 0$
$x = 0$
$x = \pi$

$2\cos(x) = -2$
$\cos(x) = -1$
$x = \pi$



$\cos^2(x) + 2\cos(x) =$

$\cos^2(0) + 2\cos(0) = 3$

$\pi \qquad f(\pi) = -1$

$2\pi \qquad f(2\pi) = 3$

$(\pi, -1)$ is the min because $f(\pi)$ has the lowest value in the 3 x's $x = 0, x = \pi, x = 2\pi$.

$x=0$ is one of the critical points and end points. $x=\pi$ is a critical point. $x = 2\pi$ is the end point.

c) $f'(x) = -2\sin x(\cos x + 1)$

$f''(x) = -2[\cos x(\cos x + 1) + \sin x(-\sin x)]$

$= -2[\cos^2 x + \cos x - \sin^2 x]$

$= -2[\cos^2 x + \cos x - [1 - \cos^2 x]]$

$= -2[2\cos^2 x + \cos x - 1]$

$= -2[(2\cos x - 1)(\cos x + 1)]$

so $f''(x) = 0 \Rightarrow$

$2\cos x - 1 = 0$ or $\cos x = \frac{1}{2}$

$\cos x + 1 = 0$ or $\cos x = -1$

$\sin^2 x + \cos^2 x = 1$